Module 07 Quiz: t Confidence Intervals

Problem 1

How does increasing the degrees of freedom affect the critical t-value?
A) It increases the critical t-value
B) It decreases the critical t-value
C) It has no effect
D) It depends on the sample size

Answer: B) It decreases the critical t-value
Explanation: As degrees of freedom increase, the t-distribution approaches the normal distribution, reducing the critical t-value for a given α.

Problem 2.
If the sample size is 25, what are the degrees of freedom for the t-distribution?
A) 24
B) 25
C) 26
D) Depends on the data

Answer: A) 24
Explanation: Degrees of freedom (df) = n - 1 = 25 - 1 = 24.

Problem 3

For a 99% confidence interval with df = 20, what is the critical t-value?

A) 1.725
B) 2.086
C) 2.845
D) 3.850

Answer: C) 2.845
Explanation: For df = 20 and $\alpha/2 = 0.005$ (two-tailed), the critical t-value is ± 2.845.

Problem 4

Which of the following does NOT affect the critical t-value?
A) Degrees of freedom
B) Significance level (α)
C) Sample mean
D) One-tailed vs. two-tailed test

Answer: C) Sample mean
Explanation: The critical t-value depends on df, α, and test type—not the sample mean.

Problem 5

What assumption is required for a valid t-confidence interval for the mean?

A) The population is normally distributed
B) The sample size is greater than 30
C) The population standard deviation is known
D) The data is categorical

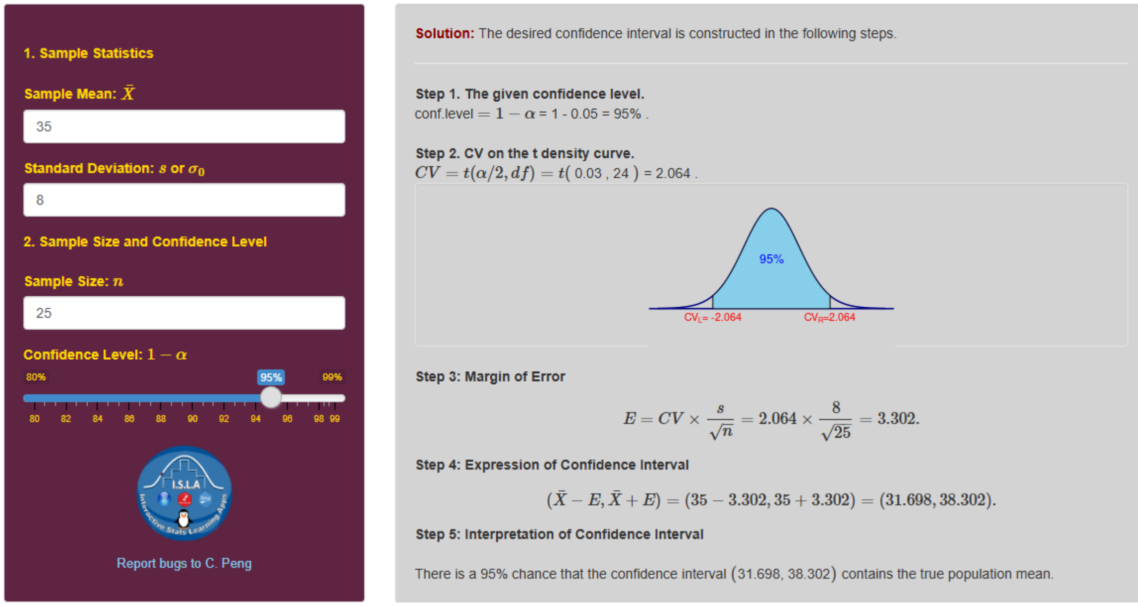
Answer: A) The population is normally distributed
Explanation: The t-interval assumes the population is approximately normal, especially for small samples. For large samples ($n > 30$), the Central Limit Theorem relaxes this requirement.

Problem 6

A transportation researcher wants to estimate the average commute time for workers in a city. A sample of 25 commuters has a mean commute time of 35 minutes with a standard deviation of 8 minutes. What is the 95% t-confidence interval for the true mean commute time? (*Hint: choose the answer closest to yours*)

A) (31.7, 38.3) minutes
B) (32.4, 37.6) minutes
C) (30.2, 39.8) minutes
D) (33.5, 36.5) minutes

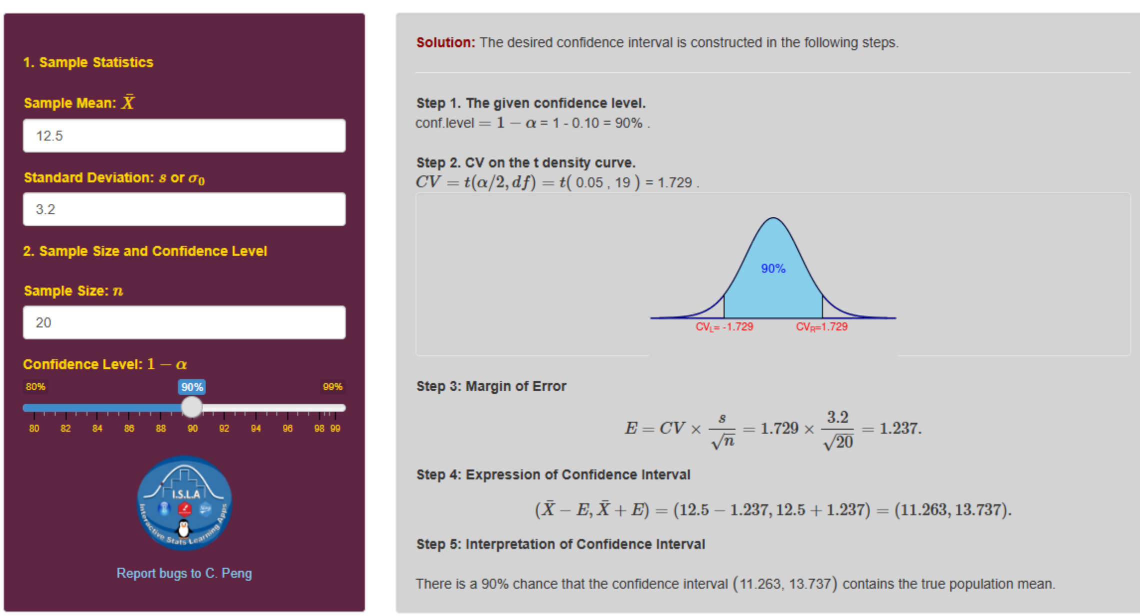
Answer: A) (31.7, 38.3) minutes



Problem 7

A coffee shop owner samples 20 customers and finds an average spending of \$12.50 per visit with a sample standard deviation of \$3.20. The 90% t-confidence interval for the true mean spending is: (*Hint: choose the answer closest to yours*)

A) (\$11.26, \$13.74)
B) (\$10.95, \$14.05)
C) (\$11.76, \$13.24)
D) (\$12.10, \$12.90)
Answer: A) (\$11.26, \$13.44)



Problem 8

A study on sleep habits surveys 15 college students, finding an average of 6.5 hours of sleep per night with a standard deviation of 1.2 hours. The 99% t-confidence interval for the mean sleep time is:

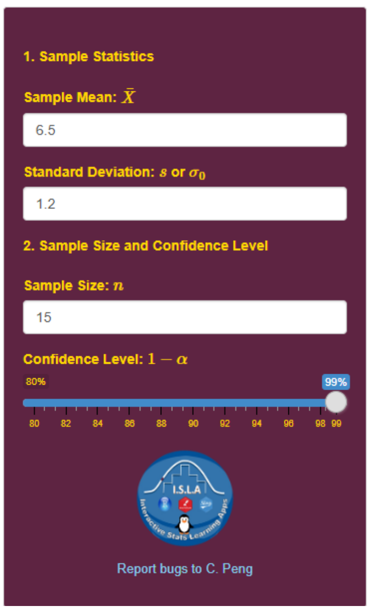
A). $15 \pm 2.624 \times \frac{1.2}{15}$

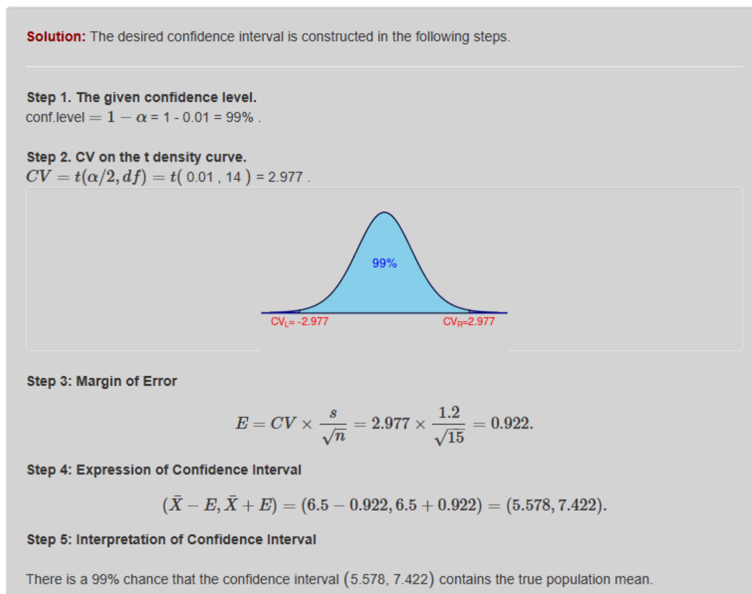
B). $15 \pm 0.99 \times \frac{1.2}{\sqrt{15}}$

C). $15 \pm 2.977 \times \frac{1.2}{\sqrt{15}}$

D). $15 \pm 2.977 \times \frac{1.2}{\sqrt{14}}$

Answer: C)



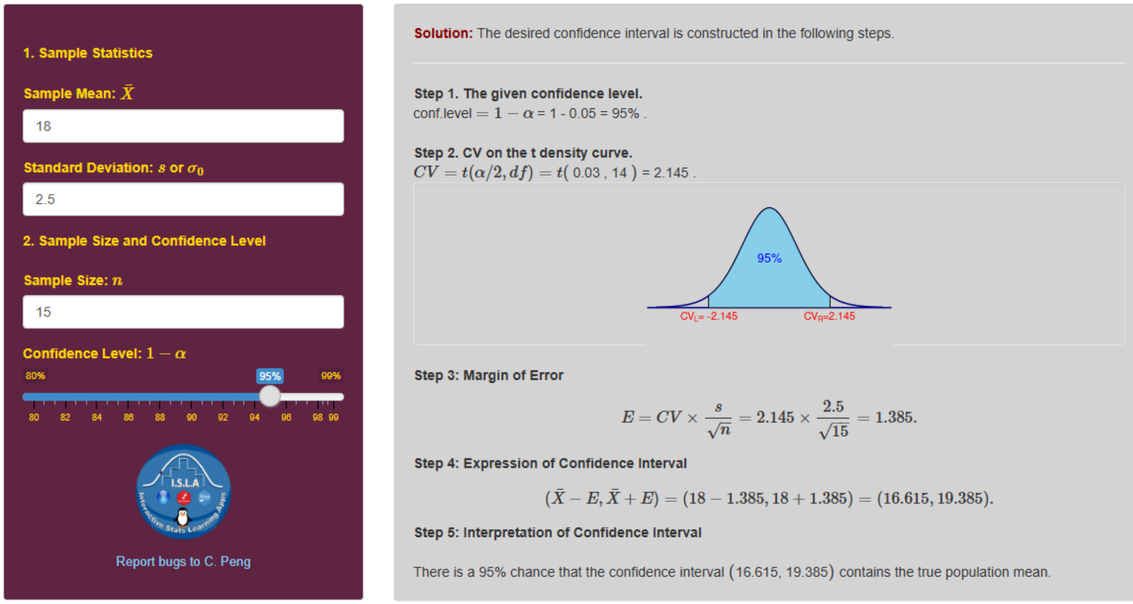


Problem 9

A tech company tests 15 smartphones and records an average battery life of 18 hours with a standard deviation of 2.5 hours. The 95% t-confidence interval for the true mean battery life is:

A). $18 \pm 2.624 \times 2.5/\sqrt{15}$
B). $18 \pm 2.145 \times 2.5/\sqrt{15}$
C). $18 \pm 1.761 \times 2.5/\sqrt{14}$
D). $18 \pm 2.145 \times 2.5/\sqrt{14}$

Answer: B)

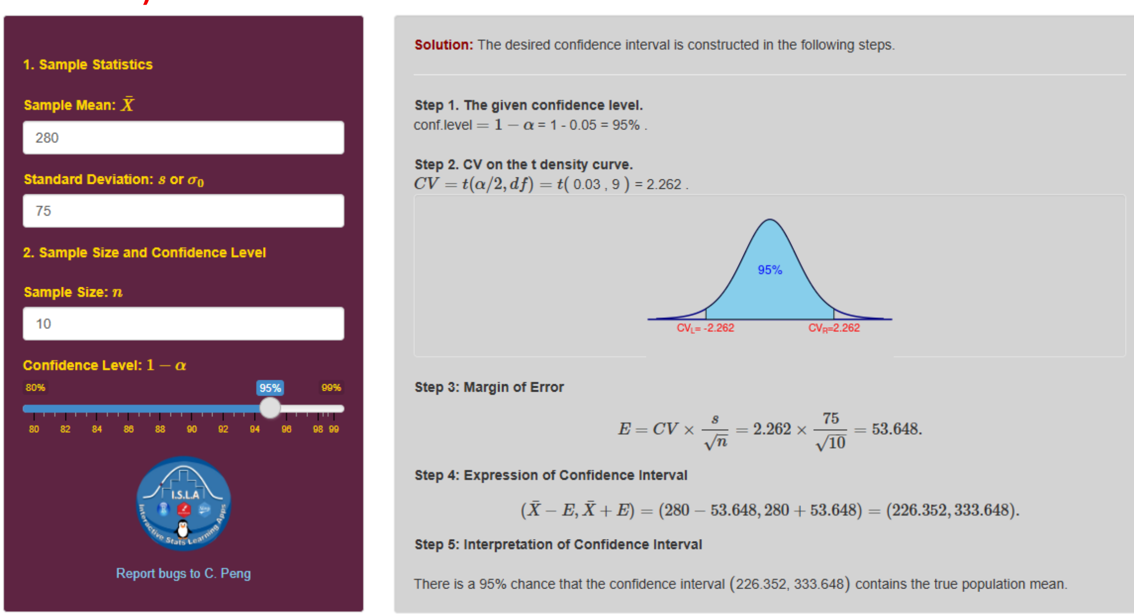


Problem 10

A university surveys 10 students and finds an average textbook expenditure of \$280 per semester with a standard deviation of \$75. The 95% t-confidence interval for the true mean expenditure is:

A). $280 \pm 2.262 \times 75/\sqrt{10}$
B). $280 \pm 2.281 \times 75/\sqrt{10}$
C). $280 \pm 2.262 \times 75/\sqrt{9}$
D). $280 \pm 0.95 \times 75/\sqrt{9}$

Answer: A)

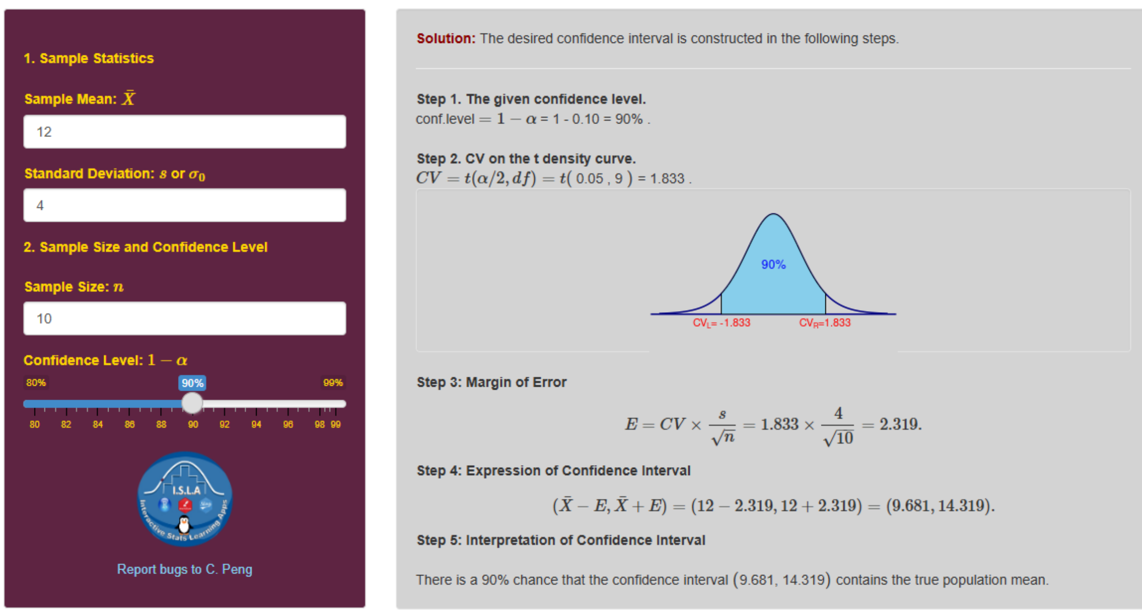


Problem 11

An agricultural study examines the effect of a new fertilizer on 10 plots of land, finding an average yield increase of 12 bushels per acre with a variance of 16 bushels. The 90% t-confidence interval for the true mean yield increase is:

A). $12 \pm 1.833 \times 4/\sqrt{10}$
B). $12 \pm 1.833 \times 16/\sqrt{10}$
C). $12 \pm 1.383 \times 4/\sqrt{10}$
D). $12 \pm 1.383 \times 16/\sqrt{10}$

Answer: A)



Problem 12

In a study of the use of hypnosis to relieve pain, sensory ratings were measured for 16 subjects and the mean sensory ratings were found to be 7.5 and variance to be 6.25. The sensory ratings are normally distributed. The margin of error of the 95% confidence interval for the mean sensory ratings of the population is

(a) $E = t_{16,\ 0.025} \times \frac{6.25}{\sqrt{15}}$

(b) $E = t_{15,\ 0.025} \times \frac{6.25}{4}$

(c) $E = t_{16,\ 0.05} \times \frac{2.5}{4}$

(d) $E = t_{15,\ 0.025} \times \frac{2.5}{\sqrt{16}}$

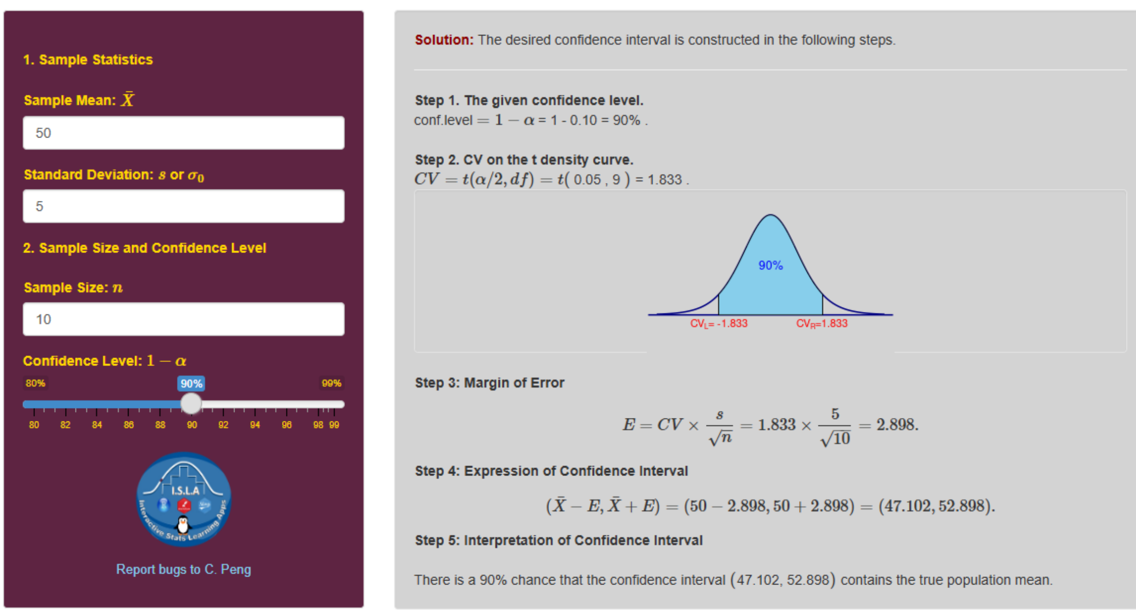
Answer D)

Problem 13

For a small sample (n = 10) from a normally distributed population, a 90% confidence interval for the mean is calculated. If the sample mean is 50 and the sample standard deviation is 5, what is the margin of error?

A). 2.898
B). 2.262
C). 3.250
D). 1.833

Answer: A) 2.821 (Using t* = 1.833 for df = 9, MOE = t* × (s/ √ n) = 1.833 × (5/ √ 10) ≈ 2.898)



Problem 14

A sample of 8 measurements has a mean of 20 and a standard deviation of 3. What is the 90% confidence interval for the true mean?

A) (17.99, 22.01)
B) (18.12, 21.88)
C) (16.34, 23.66)
D) (19.01, 20.99)

Answer: A) (18.12, 21.88)
Calculation:
$t_{df = 7, \alpha/2 = 0.05} \approx 1.895$
$E = 1.895 \times (3/\sqrt{8}) \approx 2.01$
CI = 20 ± 2.01 → (17.99, 22.01) (Closest to B, assuming rounding.)

Problem 15

If a 95% t-confidence interval for a mean is (25, 35) based on n = 16, which of the following statements is true?

A) The sample means is 30.
B) The margin of error is 10.
C) The population mean must lie between 25 and 35.
D) The standard deviation is 5.

Answer: A) The sample mean is 30.
(The midpoint of the interval is the sample mean.)