Module 05 Quiz: Sampling Distribution

Problem 1

The daily revenue at a university snack bar has been recorded for the past five years. Records indicate that the mean daily revenue is $1500, and the standard deviation is $500. The distribution is skewed to the right due to several high-volume days (football game days). Suppose that 100 days are randomly selected and the average daily revenue computed. Which of the following describes the sampling distribution of the sample mean?

A). normally distributed with a mean of $1500 and a standard deviation of $500
B). normally distributed with a mean of $1500 and a standard deviation of $50
C). normally distributed with a mean of $150 and a standard deviation of $50
D). skewed to the right with a mean of $1500 and a standard deviation of $500

Answer: B) standard error $s_{\bar{X}}$ = 500/$\sqrt{100}$=50, $\bar{X} \sim N(1500, 50)$

Problem 2.
Suppose students' ages follow a skewed right distribution with a mean of 23 years old and a standard deviation of 4 years. If we randomly sample 200 students, which of the following statements about the sampling distribution of the sample mean age is incorrect?
A) The mean of sampling distribution is approximately 23 years old.
B) The standard deviation of the sampling distribution is equal to 4 years.
C) The shape of the sampling distribution is approximately normal

Answer: C). This is based on the central limit theorem. $s_{\bar{X}}$ = 4/$\sqrt{200}$=0.283, $\bar{X} \sim N(23, 0.283)$

Problem 3

A random sample of n = 600 measurements is drawn from a binomial population with probability of success .08. What is the sampling distribution of the sample proportion, $\hat{p}$.

A) N(.08; .011)
B) N(.92; .003)
C) N(.08; .003)
D) N(.92; .011)

Answer: A) $s_{\hat{p}} = \sqrt{0.08(1-0.08)/600} = 0.0111$

Problem 4

You form a distribution of the means of all samples of size 36 drawn from an infinite population that is skewed to the left. The population from which the samples are drawn has a mean of 50 and a standard deviation of 12 . Which one of the following statements is true of this distribution?

(a) $\mu_{\bar{x}} = 50, \sigma_{\bar{x}} = 12$, the sampling distribution is skewed somewhat to the left.
(b) $\mu_{\bar{x}} = 50, \sigma_{\bar{x}} = 2$, the sampling distribution is skewed somewhat to the left.
(c) $\mu_{\bar{x}} = 50, \sigma_{\bar{x}} = 12$, the sampling distribution is approximately normal.
(d) $\mu_{\bar{x}} = 50, \sigma_{\bar{x}} = 2$, the sampling distribution is approximately normal.

Answer: D)

Problem 5

An auto insurance company has 32,000 clients, and 5% of their clients submitted a claim in the past year. We will take a sample of 3,200 clients and determine how many of them have submitted a claim in the past year. What is the sampling distribution of $\hat{p}$?

a) $\hat{p}$ ~ N(3200, 0.2)
b) $\hat{p}$ ~N(160, 152)
c) $\hat{p}$ ~N(0.05,0.003852)
d) Can not be determined

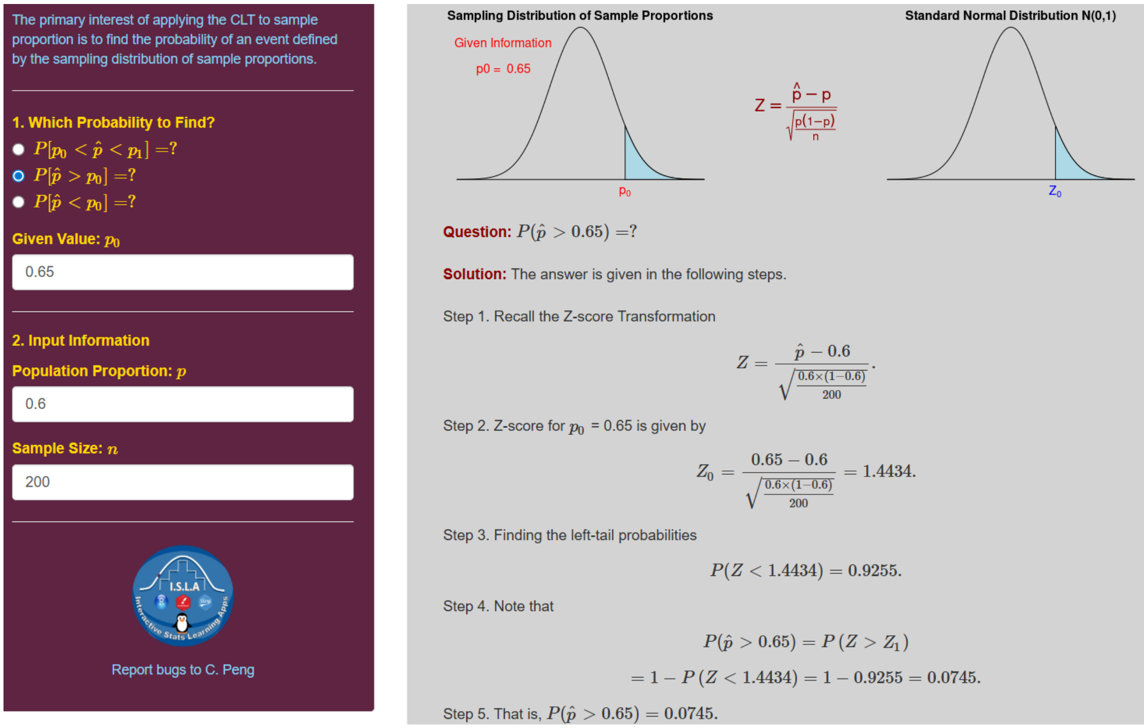
Answer: C) $\sigma_{\hat{p}} = \sqrt{\frac{0.05(1-0.05)}{3200}} = 0.00385$

Problem 6

A survey indicates that 60% of adults in a certain city support a new public transportation project. If a random sample of 200 adults is taken, what is the probability that the sample proportion supporting the project is greater than 65%?

a) Approximately 0.0745
b) Approximately 0.1151
c) Approximately 0.2611
d) Approximately 0.3821

Answer: A) $\sigma_{\hat{p}} = \sqrt{\frac{0.6(1-0.6)}{200}} = 0.0346$, by CLT $\hat{p} \sim N(0.6, 0.0346)$. Next, we find z-score of 0.65: zo = (0.65-0.6)/0.0346 = 1.445. P(Z >1.445) = 0.0742



Problem 7

In a large population, 40% of individuals have a certain genetic trait. A random sample of 400 individuals is selected. Which of the following statements about the sampling distribution of the sample proportion is **incorrect**?

a) The mean of the sampling distribution is 0.40.
b) The standard deviation of the sampling distribution is approximately 0.0245.
c) The shape of the sampling distribution is approximately normal.
d) The standard deviation of the sampling distribution is larger than the standard deviation of the population.

Answer: D) Population standard deviation $\sqrt{0.4(1-0.4)} = \sqrt{0.24}$, while the standard deviation of the sampling distribution is $\sqrt{0.4(1-0.4)/400} = \sqrt{0.24}/20$. Therefore, the standard deviation of the sampling distribution is **smaller** than the standard deviation of the population.

Problem 8

A population has a mean of 50 and a standard deviation of 8. A random sample of size 64 is taken. Which of the following statements about the sampling distribution of the sample mean is **incorrect**?

a) The mean of the sampling distribution is 50.
b) The standard deviation of the sampling distribution is 1.

c) The sampling distribution is approximately normal.
d) The standard deviation of the sampling distribution is 8.

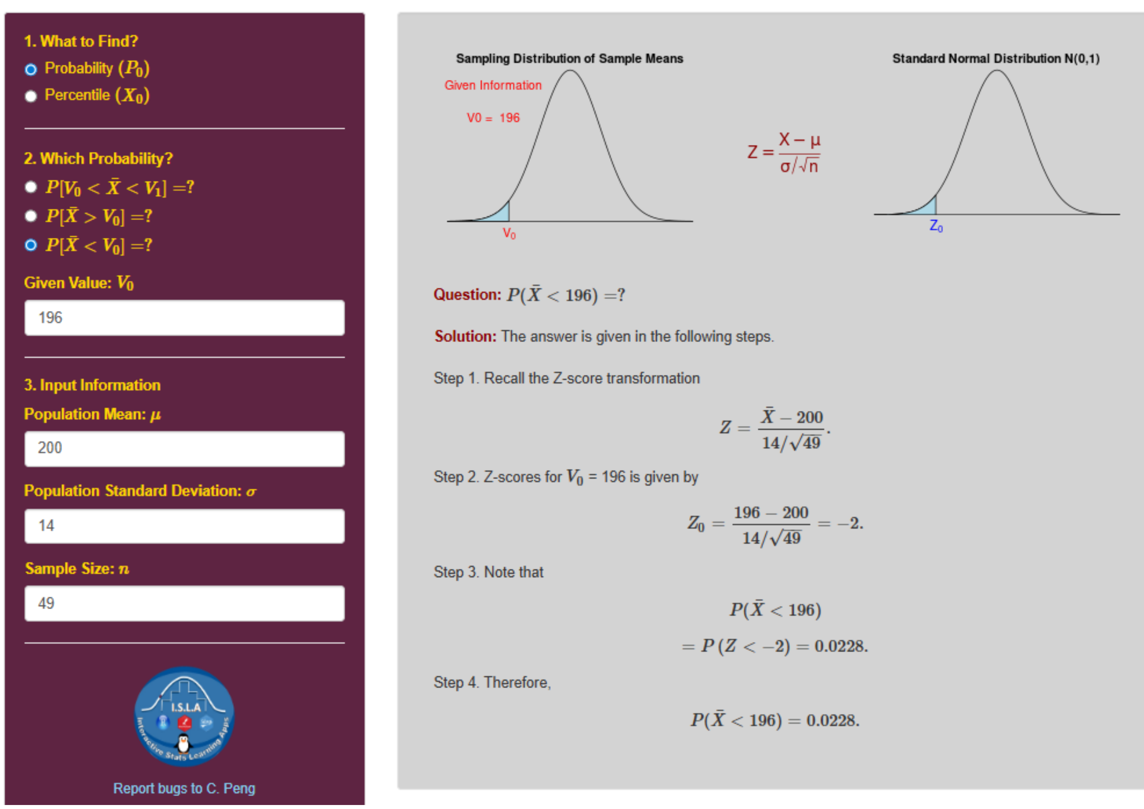
Answer: D) The standard deviation of the sampling distribution is $8/\sqrt{50} \approx 1.131$

Problem 9

A sample of size 49 is taken from a population with a mean of 200 and a standard deviation of 14. What is the probability that the sample mean will be less than 196?

a) 0.0013
b) 0.0228
c) 0.0475
d) 0.1587

Answer: B) Using the CLT, $\bar{X} \sim N\left(200, \frac{14}{\sqrt{49}}\right) = N(200, 2)$. Zo = (196-200)/2 = -2 ➔ P(Z < -2) = 0.0228

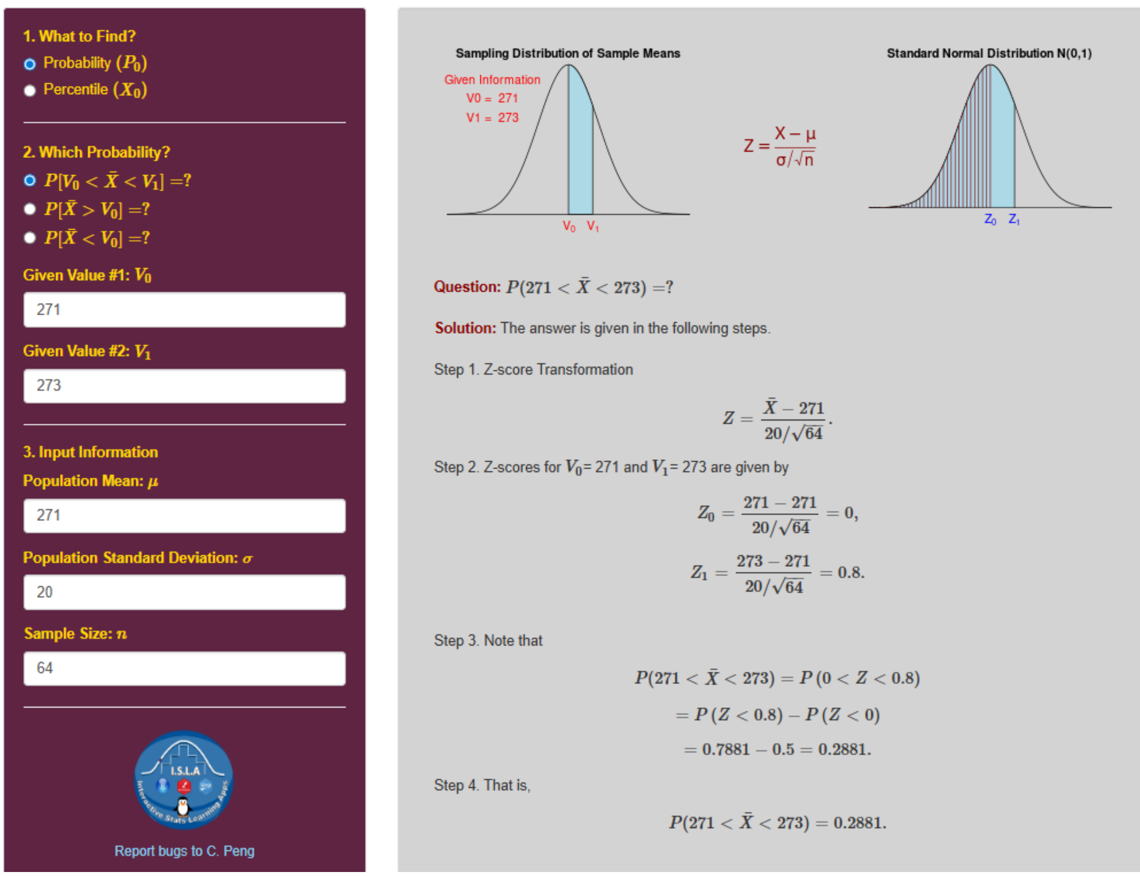


Problem 10

The lengths of pregnancies are normally distributed with a mean of 271 days and a standard deviation of 20 days. If 64 women are randomly selected, find the probability that they have a mean pregnancy between 271 days and 273 days.

A). 0.2119
B). 0.5517
C). 0.2881
D). 0.7881

Answer: C). First $\bar{X} \sim N\left(271, \frac{20}{\sqrt{64}}\right) = N(271, 2.5)$, Two corresponding z-scores are: z1 = (271-271/2.5 = 0, z2 = (273-271)/2.5 = 0.8. ➔ P(Z < 0.8) – P(Z < 0) = 0.7881 – 0.5 = 0.2881

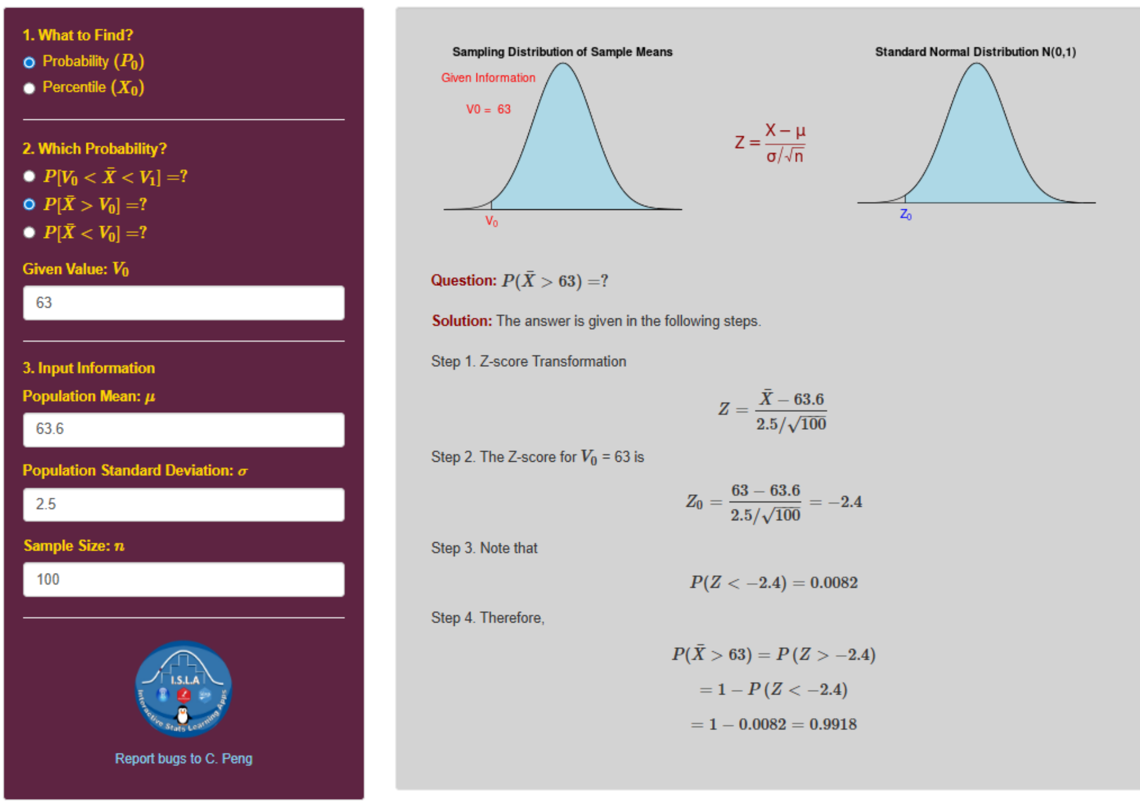


Problem 11

Assume that the heights of women are normally distributed with a mean of 63.6 inches and a standard deviation of 2.5 inches. If 100 women are randomly selected, find the probability that they have a mean height greater than 63.0 inches.

A). 0.2881
B). 0.0082
C). 0.9918
D). 0.8989

Answer: C). First $\bar{X} \sim N\left(63.6, \frac{2.5}{\sqrt{100}}\right) = N(63.6, 0.25)$, the z-scores is: z1 = (63.0 – 63.6/0.25 = -2.4, ➔ P(Z < -2.4) = 0.0082 ➔ P($\bar{X} > 63.0$) = 1 – 0.0082 = 0.9918.

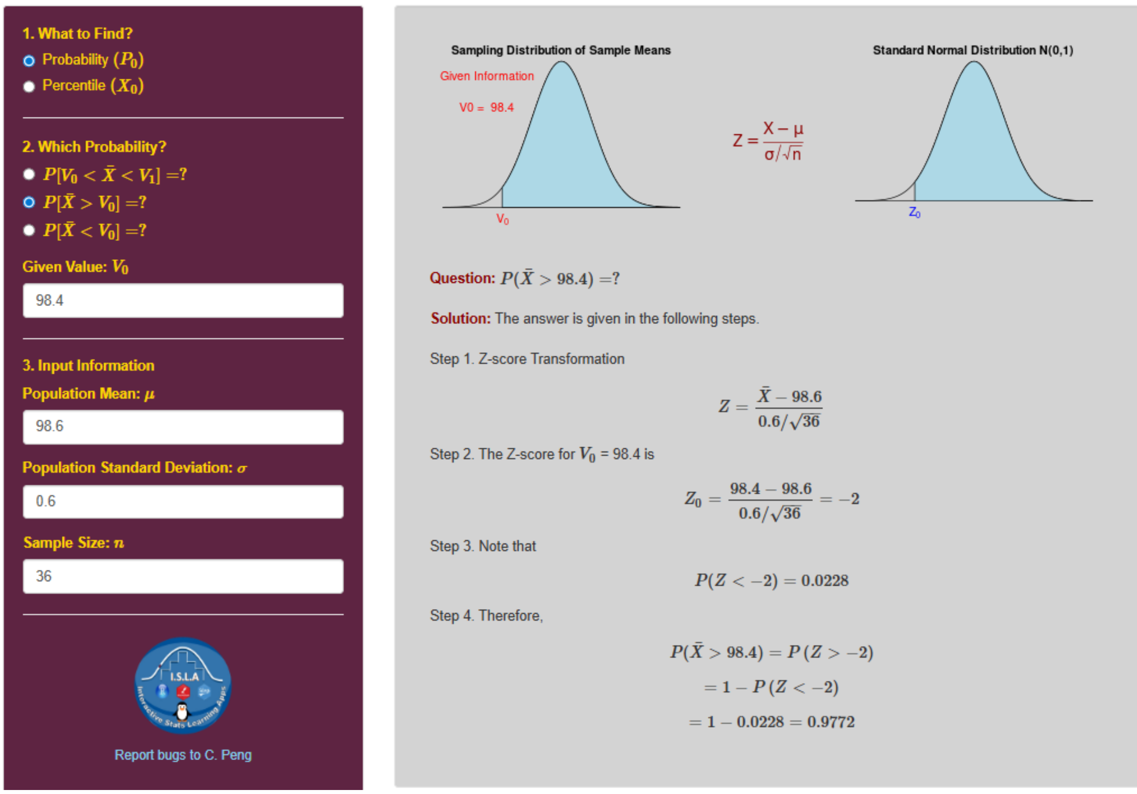


Problem 12

The body temperatures of adults are normally distributed with a mean of 98.6° F and a standard deviation of 0.60° F. If 36 adults are randomly selected, find the probability that their mean body temperature is greater than 98.4° F.

A). 0.9360
B). 0.0228
C). 0.8188
D). 0.9772

Answer D). First $\bar{X} \sim N\left(98.6, \frac{0.6}{\sqrt{36}}\right) = N(98.6, 0.1)$, the z-scores is: z1 = (98.4 – 98.6)/0.1 = -2, ➔ P(Z > -2)=1 – P(Z<2) = 1 – 0.02275 = 0.9772.



Problem 13

A random sample of n = 300 measurements is drawn from a binomial population with probability of success .43. Give the mean and the standard deviation of the sampling distribution of the sample proportion, $\hat{p}$.

A). N(.57; .029)
B). N(.43; .014)
C). N(.57; .014)
D). N(.43; .029)

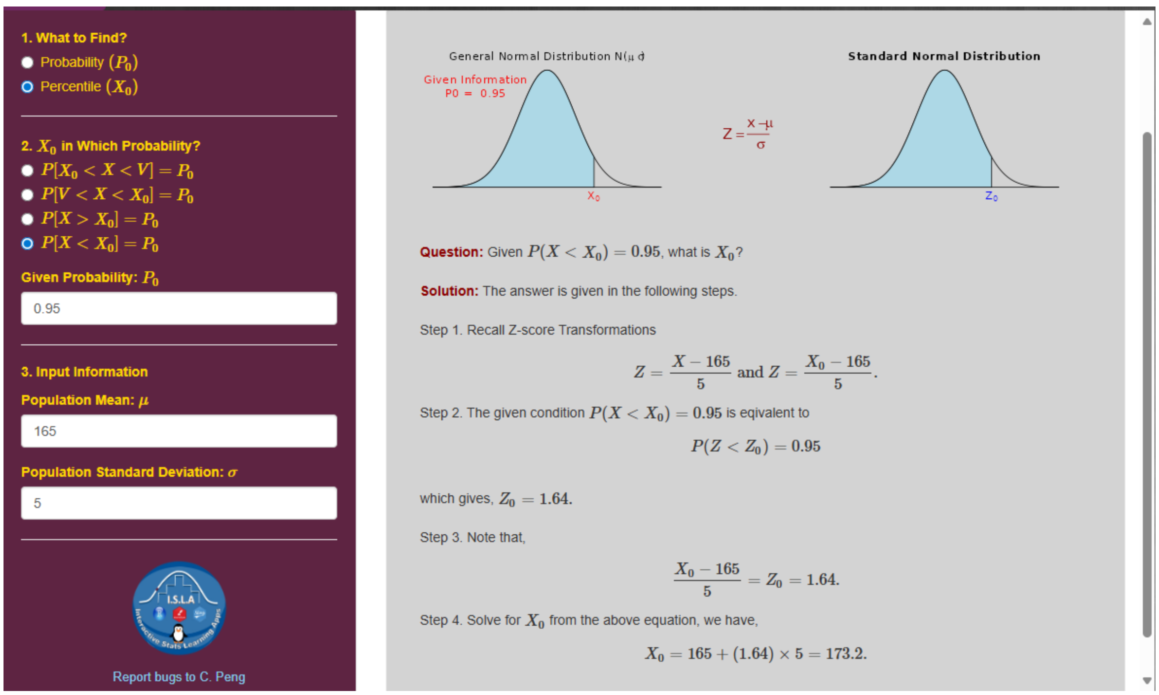
Answer D). $\sigma_{\hat{p}} = \sqrt{0.43(1-0.43)/300} \approx 0.0285.$

Problem 14.

The heights of adult women in a city are normally distributed with a mean of 165 cm and a standard deviation of 5 cm. What is the height that 95% of women are shorter than?

a) 173.25 cm
b) 171.45 cm
c) 175.00 cm
d) 174.60 cm

Answer: A.



Problem 15.

The test scores in a large class are normally distributed with a mean of 75 and a standard deviation of 10. What is the probability that a randomly selected student scored between 70 and 85?

a) 0.5328
b) 0.6826
c) 0.7745
d) 0.3413

Answer: A

1. What to Find?

- (selected) Probability (P_0)
- Percentile (X_0)

2. Which Probability?

- (selected) $P[V_0 < X < V_1] = ?$
- $P[X > V_0] = ?$
- $P[X < V_0] = ?$

Given Value #1: V_0

70

Given Value #2: V_1

85

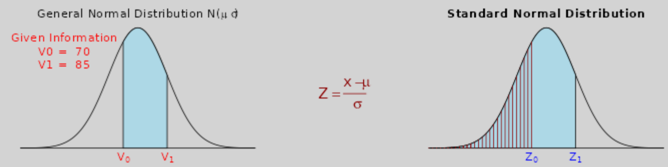
3. Input Information

Population Mean: μ

75

Population Standard Deviation: σ

10



Question: $P(70 < X < 85) = ?$

Solution: The answer is given in the following steps.

Step 1. Recall Z-score Transformation

$$Z = \frac{X - 75}{10}.$$

Step 2. Z-scores for $V_0 = 70$ and $V_1 = 85$ are given respectively by

$$Z_0 = \frac{70 - 75}{10} = -0.5 \text{ and } Z_1 = \frac{85 - 75}{10} = 1.$$

Step 3. The two left-tail Probabilities are

$$P(Z < 1) = 0.8413 \text{ and } P(Z < -0.5) = 0.3085.$$

Step 4. Note that

$$\begin{aligned} P(Z_0 < Z < Z_1) &= P(Z < Z_1) - P(Z < Z_0) \\ &= P(Z < 1) - P(Z < -0.5) \\ &= 0.8413 - 0.3085 = 0.5328. \end{aligned}$$