

## Week 05 Quiz: Sampling Distribution

### Problem 1

The daily revenue at a university snack bar has been recorded for the past five years. Records indicate that the mean daily revenue is \$1500, and the standard deviation is \$500. The distribution is skewed to the right due to several high-volume days (football game days). Suppose that 100 days are randomly selected and the average daily revenue computed. Which of the following describes the sampling distribution of the sample mean?

- A). normally distributed with a mean of \$1500 and a standard deviation of \$500
- B). normally distributed with a mean of \$1500 and a standard deviation of \$50
- C). normally distributed with a mean of \$150 and a standard deviation of \$50
- D). skewed to the right with a mean of \$1500 and a standard deviation of \$500

**Answer: B)** standard error  $s_{\bar{x}} = 500/\sqrt{100}=50$ ,  $\bar{X} \sim N(1500, 50)$

### Problem 2.

Suppose students' ages follow a skewed right distribution with a mean of 23 years old and a standard deviation of 4 years. If we randomly sample 200 students, which of the following statements about the sampling distribution of the sample mean age is incorrect?

- A) The mean of sampling distribution is approximately 23 years old.
- B) The standard deviation of the sampling distribution is equal to 4 years.
- C) The shape of the sampling distribution is approximately normal

**Answer: C).** This is based on the central limit theorem.  $s_{\bar{x}} = 4/\sqrt{200}=0.283$ ,  $\bar{X} \sim N(23, 0.283)$

### Problem 3

A random sample of  $n = 600$  measurements is drawn from a binomial population with probability of success .08. What is the sampling distribution of the sample proportion,  $\hat{p}$ .

- A)  $N(.08; .011)$
- B)  $N(.92; .003)$
- C)  $N(.08; .003)$
- D)  $N(.92; .011)$

**Answer: A)**  $s_{\hat{p}} = \sqrt{0.08(1 - 0.08)/600} = 0.0111$

#### Problem 4

You form a distribution of the means of all samples of size 36 drawn from an infinite population that is skewed to the left. The population from which the samples are drawn has a mean of 50 and a standard deviation of 12 . Which one of the following statements is true of this distribution?

- (a)  $\mu_{\bar{x}} = 50, \sigma_{\bar{x}} = 12$ , the sampling distribution is skewed somewhat to the left.
- (b)  $\mu_{\bar{x}} = 50, \sigma_{\bar{x}} = 2$ , the sampling distribution is skewed somewhat to the left.
- (c)  $\mu_{\bar{x}} = 50, \sigma_{\bar{x}} = 12$ , the sampling distribution is approximately normal.
- (d)  $\mu_{\bar{x}} = 50, \sigma_{\bar{x}} = 2$ , the sampling distribution is approximately normal.

**Answer: D)**

#### Problem 5

An auto insurance company has 32,000 clients, and 5% of their clients submitted a claim in the past year. We will take a sample of 3,200 clients and determine how many of them have submitted a claim in the past year. What is the sampling distribution of  $\hat{p}$  ?

- a)  $\hat{p} \sim N(3200, 0.2)$
- b)  $\hat{p} \sim N(160, 152)$
- c)  $\hat{p} \sim N(0.05, 0.003852)$
- d) Can not be determined

**Answer: C)**  $\sigma_{\hat{p}} = \sqrt{\frac{0.05(1-0.05)}{3200}} = 0.00385$

#### Problem 6

A survey indicates that 60% of adults in a certain city support a new public transportation project. If a random sample of 200 adults is taken, what is the probability that the sample proportion supporting the project is greater than 65%?

- a) Approximately 0.0745
- b) Approximately 0.1151
- c) Approximately 0.2611
- d) Approximately 0.3821

**Answer: A)**  $\sigma_{\hat{p}} = \sqrt{\frac{0.6(1-0.6)}{200}} = 0.0346$ , by CLT  $\hat{p} \sim N(0.6, 0.0346)$ . Next, we find z-score of 0.65:  $z_0 = (0.65-0.6)/0.0346 = 1.445$ .  $P(Z > 1.445) = 0.0742$

The primary interest of applying the CLT to sample proportion is to find the probability of an event defined by the sampling distribution of sample proportions.

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**1. Which Probability to Find?**

- ☐  $P[p_0 < \hat{p} < p_1] = ?$
- ☒  $P[\hat{p} > p_0] = ?$
- ☐  $P[\hat{p} < p_0] = ?$

**Given Value:  $p_0$**


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**2. Input Information**

**Population Proportion:  $p$**

**Sample Size:  $n$**

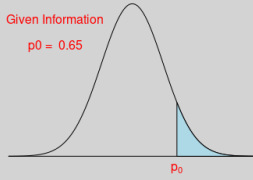
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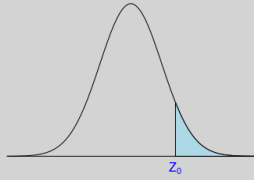
**Sampling Distribution of Sample Proportions**

Given Information  
 $p_0 = 0.65$



$p_0$

**Standard Normal Distribution  $N(0,1)$**



$Z_0$

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

**Question:**  $P(\hat{p} > 0.65) = ?$

**Solution:** The answer is given in the following steps.

Step 1. Recall the Z-score Transformation

$$Z = \frac{\hat{p} - 0.6}{\sqrt{\frac{0.6 \times (1-0.6)}{200}}}$$

Step 2. Z-score for  $p_0 = 0.65$  is given by

$$Z_0 = \frac{0.65 - 0.6}{\sqrt{\frac{0.6 \times (1-0.6)}{200}}} = 1.4434.$$

Step 3. Finding the left-tail probabilities

$$P(Z < 1.4434) = 0.9255.$$

Step 4. Note that

$$P(\hat{p} > 0.65) = P(Z > Z_1)$$

$$= 1 - P(Z < 1.4434) = 1 - 0.9255 = 0.0745.$$

Step 5. That is,  $P(\hat{p} > 0.65) = 0.0745$ .

## Problem 7

In a large population, 40% of individuals have a certain genetic trait. A random sample of 400 individuals is selected. Which of the following statements about the sampling distribution of the sample proportion is **incorrect**?

- a) The mean of the sampling distribution is 0.40.
- b) The standard deviation of the sampling distribution is approximately 0.0245.
- c) The shape of the sampling distribution is approximately normal.
- d) The standard deviation of the sampling distribution is larger than the standard deviation of the population.

**Answer: D)** Population standard deviation  $\sqrt{0.4(1-0.4)} = \sqrt{0.24}$ , while the standard deviation of the sampling distribution is  $\sqrt{0.4(1-0.4)/400} = \sqrt{0.24}/20$ . Therefore, the standard deviation of the sampling distribution is **smaller** than the standard deviation of the population.

## Problem 8

A population has a mean of 50 and a standard deviation of 8. A random sample of size 64 is taken. Which of the following statements about the sampling distribution of the sample mean is **incorrect**?

- a) The mean of the sampling distribution is 50.
- b) The standard deviation of the sampling distribution is 1.

- c) The sampling distribution is approximately normal.  
d) The standard deviation of the sampling distribution is 8.

**Answer: D)** The standard deviation of the sampling distribution is  $8/\sqrt{50} \approx 1.131$

### Problem 9

A sample of size 49 is taken from a population with a mean of 200 and a standard deviation of 14. What is the probability that the sample mean will be less than 196?

- a) 0.0013  
b) 0.0228  
c) 0.0475  
d) 0.1587

**Answer: B)** Using the CLT,  $\bar{X} \sim N\left(200, \frac{14}{\sqrt{49}}\right) = N(200, 2)$ .  $Z_0 = (196-200)/2 = -2 \rightarrow P(Z < -2) = 0.0228$

**1. What to Find?**

- ☒ Probability ( $P_0$ )
- ☐ Percentile ( $X_0$ )

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**2. Which Probability?**

- ☐  $P[V_0 < \bar{X} < V_1] = ?$
- ☐  $P[\bar{X} > V_0] = ?$
- ☒  $P[\bar{X} < V_0] = ?$

Given Value:  $V_0$


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**3. Input Information**

Population Mean:  $\mu$

Population Standard Deviation:  $\sigma$

Sample Size:  $n$

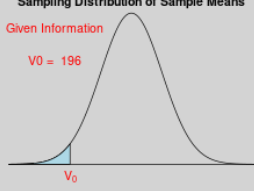


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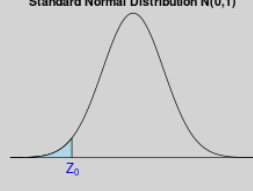
**Sampling Distribution of Sample Means**

Given Information

$V_0 = 196$



**Standard Normal Distribution N(0,1)**



$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

**Question:**  $P(\bar{X} < 196) = ?$

**Solution:** The answer is given in the following steps.

Step 1. Recall the Z-score transformation

$$Z = \frac{\bar{X} - 200}{14/\sqrt{49}}$$

Step 2. Z-scores for  $V_0 = 196$  is given by

$$Z_0 = \frac{196 - 200}{14/\sqrt{49}} = -2.$$

Step 3. Note that

$$P(\bar{X} < 196) = P(Z < -2) = 0.0228.$$

Step 4. Therefore,

$$P(\bar{X} < 196) = 0.0228.$$

## Problem 10

The lengths of pregnancies are normally distributed with a mean of 271 days and a standard deviation of 20 days. If 64 women are randomly selected, find the probability that they have a mean pregnancy between 271 days and 273 days.

- A). 0.2119
- B). 0.5517
- C). 0.2881
- D). 0.7881

**Answer: C).** First  $\bar{X} \sim N\left(271, \frac{20}{\sqrt{64}}\right) = N(271, 2.5)$ , Two corresponding z-scores are:  $z_1 = (271 - 271)/2.5 = 0$ ,  $z_2 = (273 - 271)/2.5 = 0.8$ .  $\rightarrow P(Z < 0.8) - P(Z < 0) = 0.7881 - 0.5 = 0.2881$

**1. What to Find?**

- ☒ Probability ( $P_0$ )
- ☐ Percentile ( $X_0$ )

**2. Which Probability?**

- ☒  $P[V_0 < \bar{X} < V_1] = ?$
- ☐  $P[\bar{X} > V_0] = ?$
- ☐  $P[\bar{X} < V_0] = ?$

**Given Value #1:  $V_0$** 

271

**Given Value #2:  $V_1$** 

273

**3. Input Information**

**Population Mean:  $\mu$** 


271

**Population Standard Deviation:  $\sigma$** 

20

**Sample Size:  $n$** 

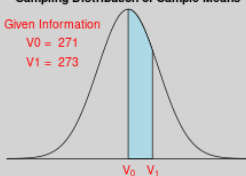
64

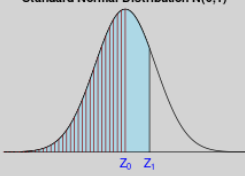


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**Sampling Distribution of Sample Means**

Given Information  
 $V_0 = 271$   
 $V_1 = 273$



**Standard Normal Distribution  $N(0,1)$** 

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

**Question:**  $P(271 < \bar{X} < 273) = ?$

**Solution:** The answer is given in the following steps.

Step 1. Z-score Transformation
$$Z = \frac{\bar{X} - 271}{20/\sqrt{64}}$$

Step 2. Z-scores for  $V_0 = 271$  and  $V_1 = 273$  are given by
$$Z_0 = \frac{271 - 271}{20/\sqrt{64}} = 0,$$
$$Z_1 = \frac{273 - 271}{20/\sqrt{64}} = 0.8.$$

Step 3. Note that
$$P(271 < \bar{X} < 273) = P(0 < Z < 0.8)$$
$$= P(Z < 0.8) - P(Z < 0)$$
$$= 0.7881 - 0.5 = 0.2881.$$

Step 4. That is,
$$P(271 < \bar{X} < 273) = 0.2881.$$

### Problem 11

Assume that the heights of women are normally distributed with a mean of 63.6 inches and a standard deviation of 2.5 inches. If 100 women are randomly selected, find the probability that they have a mean height greater than 63.0 inches.

- A). 0.2881
- B). 0.0082
- C). 0.9918
- D). 0.8989

**Answer: C).** First  $\bar{X} \sim N\left(63.6, \frac{2.5}{\sqrt{100}}\right) = N(63.6, 0.25)$ , the z-scores is:  $z_1 = (63.0 - 63.6)/0.25 = -2.4$ ,  $\rightarrow P(Z < -2.4) = 0.0082 \rightarrow P(\bar{X} > 63.0) = 1 - 0.0082 = 0.9918$ .

1. What to Find?

☒ Probability ( $P_0$ )

☐ Percentile ( $X_0$ )

2. Which Probability?

☐  $P[V_0 < \bar{X} < V_1] = ?$

☒  $P[\bar{X} > V_0] = ?$

☐  $P[\bar{X} < V_0] = ?$


Given Value:  $V_0$

3. Input Information

Population Mean:  $\mu$

Population Standard Deviation:  $\sigma$

Sample Size:  $n$

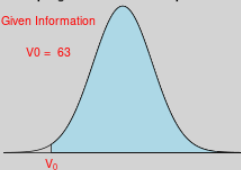


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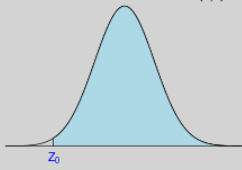
Sampling Distribution of Sample Means

Given Information

$V_0 = 63$



Standard Normal Distribution  $N(0,1)$



$$Z = \frac{X - \mu}{\sigma/\sqrt{n}}$$

Question:  $P(\bar{X} > 63) = ?$

Solution: The answer is given in the following steps.

Step 1. Z-score Transformation

$$Z = \frac{\bar{X} - 63.6}{2.5/\sqrt{100}}$$

Step 2. The Z-score for  $V_0 = 63$  is

$$Z_0 = \frac{63 - 63.6}{2.5/\sqrt{100}} = -2.4$$

Step 3. Note that

$$P(Z < -2.4) = 0.0082$$

Step 4. Therefore,

$$\begin{aligned} P(\bar{X} > 63) &= P(Z > -2.4) \\ &= 1 - P(Z < -2.4) \\ &= 1 - 0.0082 = 0.9918 \end{aligned}$$

### Problem 12

The body temperatures of adults are normally distributed with a mean of 98.6° F and a standard deviation of 0.60° F. If 36 adults are randomly selected, find the probability that their mean body temperature is greater than 98.4° F.

- A). 0.9360
- B). 0.0228
- C). 0.8188
- D). 0.9772

**Answer D).** First  $\bar{X} \sim N\left(98.6, \frac{0.6}{\sqrt{36}}\right) = N(98.6, 0.1)$ , the z-scores is:  $z_1 = (98.4 - 98.6)/0.1 = -2$ ,  $\rightarrow P(Z > -2) = 1 - P(Z < -2) = 1 - 0.02275 = 0.9772$ .

**1. What to Find?**

- ☒ Probability ( $P_0$ )
- ☐ Percentile ( $X_0$ )

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**2. Which Probability?**

- ☐  $P[V_0 < \bar{X} < V_1] = ?$
- ☒  $P[\bar{X} > V_0] = ?$
- ☐  $P[\bar{X} < V_0] = ?$

Given Value:  $V_0$


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**3. Input Information**

Population Mean:  $\mu$

Population Standard Deviation:  $\sigma$

Sample Size:  $n$

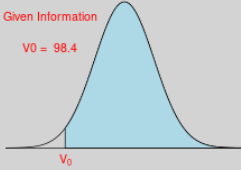


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**Sampling Distribution of Sample Means**

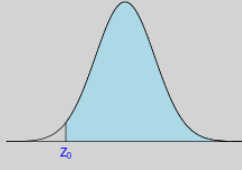
Given Information

$V_0 = 98.4$



$V_0$

**Standard Normal Distribution N(0,1)**



$Z_0$

$$Z = \frac{X - \mu}{\sigma/\sqrt{n}}$$

**Question:**  $P(\bar{X} > 98.4) = ?$

**Solution:** The answer is given in the following steps.

Step 1. Z-score Transformation

$$Z = \frac{\bar{X} - 98.6}{0.6/\sqrt{36}}$$

Step 2. The Z-score for  $V_0 = 98.4$  is

$$Z_0 = \frac{98.4 - 98.6}{0.6/\sqrt{36}} = -2$$

Step 3. Note that

$$P(Z < -2) = 0.0228$$

Step 4. Therefore,

$$\begin{aligned} P(\bar{X} > 98.4) &= P(Z > -2) \\ &= 1 - P(Z < -2) \\ &= 1 - 0.0228 = 0.9772 \end{aligned}$$

### Problem 13

A random sample of  $n = 300$  measurements is drawn from a binomial population with probability of success .43. Give the mean and the standard deviation of the sampling distribution of the sample proportion,  $\hat{p}$ .

- A).  $N(.57; .029)$
- B).  $N(.43; .014)$
- C).  $N(.57; .014)$
- D).  $N(.43; .029)$

**Answer D).**  $\sigma_{\hat{p}} = \sqrt{0.43(1 - 0.43)/300} \approx 0.0285$ .

### Problem 14.

The heights of adult women in a city are normally distributed with a mean of 165 cm and a standard deviation of 5 cm. What is the height that 95% of women are shorter than?

- a) 173.25 cm
- b) 171.45 cm
- c) 175.00 cm
- d) 174.60 cm

**Answer: A.**

**1. What to Find?**

- ☐ Probability ( $P_0$ )
- ☒ Percentile ( $X_0$ )

**2.  $X_0$  in Which Probability?**

- ☐  $P[X_0 < X < V] = P_0$
- ☐  $P[V < X < X_0] = P_0$
- ☐  $P[X > X_0] = P_0$
- ☒  $P[X < X_0] = P_0$

**Given Probability:  $P_0$**

0.95


**3. Input Information**

**Population Mean:  $\mu$**

165

**Population Standard Deviation:  $\sigma$**

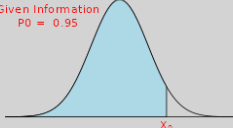
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General Normal Distribution  $N(\mu, \sigma)$

Given Information  
 $P_0 = 0.95$



$Z = \frac{X - \mu}{\sigma}$

Standard Normal Distribution



**Question:** Given  $P(X < X_0) = 0.95$ , what is  $X_0$ ?

**Solution:** The answer is given in the following steps.

Step 1. Recall Z-score Transformations

$$Z = \frac{X - 165}{5} \text{ and } Z = \frac{X_0 - 165}{5}.$$

Step 2. The given condition  $P(X < X_0) = 0.95$  is equivalent to

$$P(Z < Z_0) = 0.95$$

which gives,  $Z_0 = 1.64$ .

Step 3. Note that,

$$\frac{X_0 - 165}{5} = Z_0 = 1.64.$$

Step 4. Solve for  $X_0$  from the above equation, we have,

$$X_0 = 165 + (1.64) \times 5 = 173.2.$$

### Problem 15.

The test scores in a large class are normally distributed with a mean of 75 and a standard deviation of 10. What is the probability that a randomly selected student scored between 70 and 85?

- a) 0.5328
- b) 0.6826
- c) 0.7745
- d) 0.3413

**Answer: A**



**1. What to Find?**

- ☒ Probability ( $P_0$ )
- ☐ Percentile ( $X_0$ )

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**2. Which Probability?**

- ☒  $P[V_0 < X < V_1] = ?$
- ☐  $P[X > V_0] = ?$
- ☐  $P[X < V_0] = ?$

**Given Value #1:  $V_0$**

70

**Given Value #2:  $V_1$**

85

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**3. Input Information**

**Population Mean:  $\mu$**

75

**Population Standard Deviation:  $\sigma$**

10

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General Normal Distribution  $N(\mu, \sigma)$

Given Information  
 $V_0 = 70$   
 $V_1 = 85$

$V_0$     $V_1$

Standard Normal Distribution

$Z_0$     $Z_1$

$$Z = \frac{X - \mu}{\sigma}$$

**Question:**  $P(70 < X < 85) = ?$

**Solution:** The answer is given in the following steps.

Step 1. Recall Z-score Transformation

$$Z = \frac{X - 75}{10}$$

Step 2. Z-scores for  $V_0 = 70$  and  $V_1 = 85$  are given respectively by

$$Z_0 = \frac{70 - 75}{10} = -0.5 \text{ and } Z_1 = \frac{85 - 75}{10} = 1.$$

Step 3. The two left-tail Probabilities are

$$P(Z < 1) = 0.8413 \text{ and } P(Z < -0.5) = 0.3085.$$

Step 4. Note that

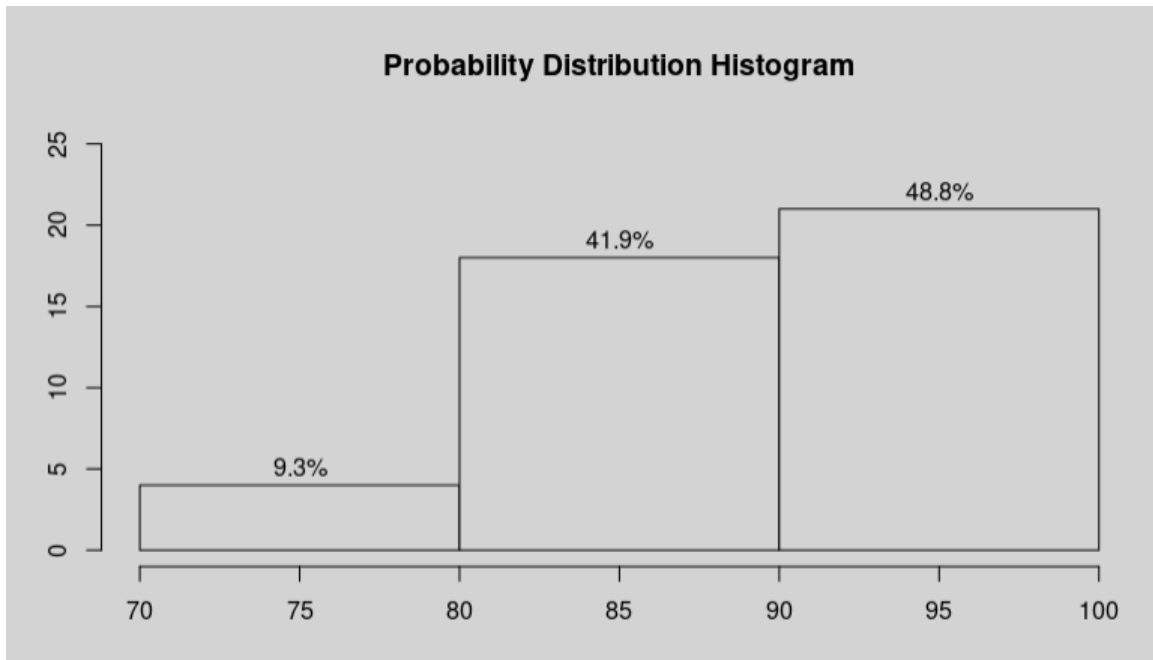
$$\begin{aligned} P(Z_0 < Z < Z_1) &= P(Z < Z_1) - P(Z < Z_0) \\ &= P(Z < 1) - P(Z < -0.5) \\ &= 0.8413 - 0.3085 = 0.5328. \end{aligned}$$

Step 5. Therefore

## Summary of Weekly Assignment #5

The class boundary is: 70,80,90,100

cut.data.freq	Freq	midpts	rel.freq	cum.freq	rel.cum.freq
[7e+01,8e+01]	4	75.00	0.09	4	0.09
(8e+01,9e+01]	18	85.00	0.42	22	0.51
(9e+01,1e+02]	21	95.00	0.49	43	1.00



### 1. Five Number Summary :

The five-number summary is used to describe the shape of the distribution of a given numerical data. It consists of five numbers: minimum data value, first quartile, median, the third quartile, and the maximum data value.

The five-number summary of this given data set is:

stats	value
Min.	70.00
1st Qu.	90.00
Median	90.00
3rd Qu.	95.00
Max.	95.00

### 2. Boxplot :

The boxplot is a geometric representation of the five-number summary. The boxplot of the given data set is given below.

