Procedures of Testing Hypothesis for the Difference of Two Population Means and Proportions

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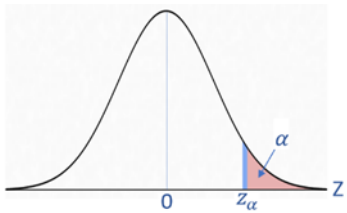

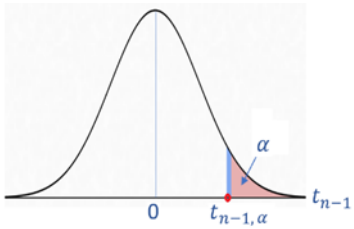

Population parameters

$\mu_1,\ \sigma_1^2,\ p_1; \mu_2, \sigma_2^2,\ p_2$

Sample statistics:

$\bar{x}_1, s_1, n_1, \hat{p}_1;\ \bar{x}_2, s_2, n_2, \hat{p}_2$

$n_1 \le 30$ or $n_2 \le 30$ normal pops.

$n_1 > 30\ \&\ n_2 > 30$

Ho: $p_1 = p_2$

$n_1 p_1 > 5\ \&\ n_1(1-p_1) > 5$

$n_2 p_2 > 5\ \&\ n_2(1-p_2) > 5$

$$\hat{p}_1 - \hat{p}_2 \to N\left(0, \sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}\right)$$

$$TS = (\hat{p}_1 - \hat{p}_2) \Big/ \sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

C.V. $Z_\alpha\ (-Z_\alpha, \pm Z_{\alpha/2})$,

$$\hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1+n_2}$$

σ_1 and σ_2 are given

$$\bar{X}_1 - \bar{X}_2 \to N\left(0, \sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}\right)$$

Replace s_1 and s_2 with σ_1 and σ_2

$$TS = \left(\bar{X}_1 - \bar{X}_2\right) \Big/ \sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$$

C.V. $t_{n_1+n_2-2,\alpha}(-t_{n_1+n_2-2,\alpha}, \pm t_{n_1+n_2-2,\alpha/2})$

$\sigma_1 = \sigma_2$ are unknown

$$\frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \to t_{n_1+n_2-2}$$

$$TS = \left(\bar{X}_1 - \bar{X}_2\right) \Big/ \sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

Decision Rule: Right-tailed test: If TS > CV, reject Ho; Otherwise, fail to reject Ho
Left-tailed test: If TS < CV, reject Ho; Otherwise, fail to reject Ho
Two-tailed test: If |TS| > |CV|, reject Ho; Otherwise, fail to reject Ho.